Decimal Numbers

A **decimal number** is a number that contains both a whole number part and a fractional part, separated by a decimal point.

The **whole number part** of a decimal number is the part to the left of the decimal point, while the **fractional part** is the part to the right:

2.35

If the whole number part of a decimal number is 0, we usually put a zero in front of the decimal point to avoid confusion. For example 0.35.

If the fractional part of a decimal number is 0, we usually omit the decimal point and write the number as a whole number. For example we write 2.0 simply as 2.
A decimal number is a number that contains both a whole number part and a fractional part, separated by a decimal point.

The whole number part of a decimal number is the part to the left of the decimal point, for example the two in two point three five.

The fractional part of a decimal number is the part to the right of the decimal point, for example the three five in two point three five.

If the whole number part of a decimal number is zero, we usually put a zero in front of the decimal point to avoid confusion. For example we would write zero point three five rather than just point three five.

If the fractional part of a decimal number is zero, we usually omit the decimal point and write the number as a whole number. For example we write two point zero simply as two.

**Rounding Decimal Numbers**

It can be useful to round decimal numbers, so that they are easier to read and understand and so that we can make estimates. For example if we say that Steve jumped 3.5m in his school’s long-jump competition, we have rounded this distance to one decimal place.

The number of digits to round to depends on the context- you might round to the nearest whole number, or to one or two (or even more) decimal places.

Something to watch out for when rounding is that you don’t round too early in your working. In fact ideally, when using a calculator you should **retain all accurate answers in your calculator as you work and not round at all until your final answer**.

It can be useful to round decimal numbers, so that they are easier to read and understand and so that we can make estimates. For example if we say that Steve jumped 3.5m in his school’s long-jump competition, we have rounded this distance to one decimal place.

The number of digits to round to depends on the context – you might round to the nearest whole number, or to one or two (or even more) decimal places.

When dealing with monetary amounts you will often need to round to the nearest cent (that is, to two decimal places) – although you may be required to round to the nearest dollar (that is, to a whole number), or to any other accuracy as specified.
Something to watch out for when rounding, particularly to a specified accuracy, is that you do not round too early on in your working.

If for example you are asked to round your final answer to two decimal places, then any working up to this point should be given to at least three or more decimal places. This enables accuracy in your final rounding.

In fact ideally, when using a calculator you should retain all accurate values in your calculator as you work and not round at all until your final answer.

Rounding to a Specified Accuracy

When rounding a decimal number to a certain number of decimal places, or to the nearest whole number, you should follow these steps:

1. Find the relevant ‘round-off place’ in your number.

   For example, when rounding 4.576 to 2 decimal places the round off place is the 2nd decimal place; 4.576

2. Determine the number one place to the right of your round-off place.

   For example, when rounding 4.576 to 2 decimal places this number is 6; 4.576

3. If this number is greater than or equal to 5 you should increase the number in the round-off place by 1, and remove all digits to the right of it. In other words, you should round up.

   If this number is less than 5, you should keep the number in the round-off place the same, and again remove all digits to the right of it. In other words, you should round down.

   For example, when rounding 4.576 to 2 decimal places you should round up (since 6 is greater than 5), so you should increase the 7 to 8 and remove all digits to the right of it; 4.58

When rounding a decimal number to a certain number of decimal places, or to the nearest whole number, you should follow a series of steps – at least until you have a good understanding of how to do it and can easily ‘see’ how to round the number.

The first step is to find the relevant ‘round-off place’ in your number; if you are rounding to $n$ decimal places then this is the $n^{th}$ decimal place, or if you are rounding to the nearest whole number this is the place to the left of the decimal point. For example, when rounding 4.576 to 2 decimal places the round off place is the 2nd decimal place, where the number seven is.

The second step is to determine the number one place to the right of this round-off place. For example, when rounding 4.576 to 2 decimal places the number to the right of the 2nd decimal place (that is, the number in the third decimal place) is six.

The third and final step is to determine whether the number found previously is greater than or equal to five. If it is, then you should increase the number in the round-off place by 1, and remove all digits to the right of it. In other words, you should round up. If it is less than five then you should keep the
number in the round-off place the same, and again remove all digits to the right of it. In other words, you should round down. For example, when rounding 4.576 to 2 decimal places you should round up, since 6 is greater than 5. So you should increase the 7 to 8 and remove all digits to the right of it, giving a final answer of 4.58.

Examples: Rounding to a Specified Accuracy

Tom has just started swimming, and manages to swim seven laps of his local swimming pool. He times that it takes him 300 seconds to do this. Assuming that Tom swims at the same speed for each lap, calculate how many seconds it takes him to swim each lap, rounded to:

a) the nearest whole number,
   Calculating 300 ÷ 7 gives 42.857142857...
   So rounded to the nearest whole number this is 43 (round up)

b) one decimal place,
   42.9 (round up)

c) two decimal places, and
   42.86 (round up)

d) three decimal places.
   42.857 (round down)

Let’s work through another example involving rounding. This example states that it takes Tom 300 seconds to swim seven laps of his local pool. The problem is to find how long it takes him to swim each individual lap, rounded to various accuracies.

Part ‘a’ asks for the time to be rounded to the nearest whole number. In order to do this we need to calculate 300 divided by 7, which gives 42.857142857, and so on forever (note that a decimal number that repeats infinitely many times like this is known as a recurring or repeating decimal). We can then round this number to the nearest whole number by looking at the first digit after the decimal point, which is 8 in this case. Since this is greater than 5 we round up to 43.

Part ‘b’ asks for the number to be rounded to one decimal place. To do this we look at the number in the second decimal place, which is 5, and since this is equal to 5 we round up to 42.9.

Part ‘c’ asks for the number to be rounded to two decimal places. To do this we look at the number in the third decimal place, which is 7, and since this is greater than 5 we round up to 42.86.

Part ‘d’ asks for the number to be rounded to three decimal places. To do this we look at the number in the fourth decimal place, which is 1, and since this is less than 5 we round down to 42.857.
Activity 1: Practice Questions

Click on the Activity 1 link in the right-hand part of this screen.

Now have a go at rounding decimal numbers on your own by working through some practice questions.

Truncation

Although rounding is the most common way of reducing the number of digits in a solution, in some cases it is not appropriate. Consider, for example, that you have $9 to buy popcorn, which costs $2.50 a bucket. A simple division calculation gives:

\[9 ÷ 2.5 = 3.6\]

However to say that this means we can buy 3.6 or 4 (when rounded to the nearest whole number) buckets of popcorn is neither practical nor correct – instead, we should say that we can buy 3.

This example is an application of truncation. In this case it is not reasonable to have any digits after the decimal point, so we truncate our original solution of 3.6 to 3. In other cases, you may be asked to truncate a figure to a particular number of decimal places. In general, truncation should only be used when rounding does not make sense (as in our popcorn example), or where specified. In all other cases, you should round numbers using the method described previously.
Although rounding is the most common way of reducing the number of digits in a solution, in some cases it is not appropriate. Consider, for example, that you have $9 to buy popcorn, which costs $2.50 a bucket. Simple division of 9 by 2.5 gives 3.6, however to say that this means we can buy 3.6 or 4 (when rounded to the nearest whole number) buckets of popcorn is neither practical nor correct—instead, we should say that we can buy 3.

This example is an application of truncation—a term for limiting the number of digits to the right of the decimal point in accordance with what is specified or appropriate. In this case it is not reasonable to have any digits after the decimal point (we can’t purchase part of a bucket), so we truncate our original solution of 3.6 to 3. In other cases, you may be asked to truncate a figure to a particular number of decimal places.

While truncating does give the same result in some cases as rounding, the two should not be confused—truncation just cuts off the number at the specified digit. Note that, in general, truncation should only be used when rounding does not make sense (as in our popcorn example), or where specified. In all other cases, you should round numbers using the method described previously.

Examples: Truncation

1. Truncate 6.35912356 to:
   a. A whole number = 6
   b. 1 decimal place = 6.3
   c. 3 decimal places = 6.359
   d. 5 decimal places = 6.35912

2. Truncate -42.5392643 to:
   a. A whole number = -42
   b. 1 decimal place = -42.5
   c. 3 decimal places = -42.539
   d. 5 decimal places = -42.53926

Let’s work through some more examples involving truncation.

Example one requires us to truncate 6.35912356 first to a whole number, which gives 6, then to 1 decimal place, which gives 6.3, then to three decimal places, which gives 6.359, and finally to five decimal places which gives 6.35912.

Example two requires us to truncate -42.5392643 first to a whole number, which gives -42, then to 1 decimal place, which gives -42.5, then to three decimal places, which gives -42.539, and finally to five decimal places which gives -42.53926.
Activity 2: Practice Questions

Click on the Activity 2 link in the right-hand part of this screen.

Now have a go at truncating decimal numbers on your own by working through some practice questions.

Converting Decimals to More Practical Forms

Sometimes when you perform a calculation you might end up with a decimal number that would be easier to interpret in another form.

For example, consider that you need to pay back an interest-free loan of $33 000, and that you will repay $12 000 per year. If you wish to determine the number of years it will take you to repay this loan, you can simply divide $33 000 by $12 000:

\[
\frac{33 \ 000}{12 \ 000} = 2.75 \text{ years}
\]

To state this in terms of years and months, consider that the ‘.75’ part of our decimal number refers to 0.75 of 12 months (i.e. 0.75 x 12 months = 9 months), and hence our final answer is 2 years and 9 months.

For example, consider that you need to pay back an interest-free loan of $33 000, and that you will repay $12 000 per year. If you wish to determine the number of years it will take you to repay this loan, you can simply divide $33 000 by $12 000 to give 2.75 years.

While 2.75 years is a valid decimal number, it is often more practical to give such a solution in terms of years and months. In order to do this, consider that the ‘.75’ part of our decimal number refers to 0.75 of one year, or 0.75 of 12 months. We can determine this by calculating 0.75 x 12 months, which gives 9 months, and hence our final answer is 2 years and 9 months.
Examples: Converting Decimals to More Practical Forms

1. Convert 2.33 years into years and months (rounded to the nearest month)
   \[0.33 \times 12 = 3.96 \approx 4\]
   So solution is 2 years and 4 months

2. Convert 3.75 days into days and hours
   There are 24 hours in a day, and \[0.75 \times 24 = 18\]
   So solution is 3 days and 18 hours

3. Convert 7.33 hours into hours and minutes (rounded to the nearest minute)
   There are 60 minutes in an hour, and \[0.33 \times 60 = 19.8 \approx 20\]
   So solution is 7 hours and 20 minutes

4. Convert 18.25 minutes into minutes and seconds
   There are 60 seconds in a minute, and \[0.25 \times 60 = 15\]
   So solution is 18 minutes and 15 seconds

Let’s work through some more examples involving converting decimals to more practical forms.

Example one requires us to convert 2.33 years into years and months, rounded to the nearest month. To do this we multiply the 0.33 part of the decimal number by 12 months to give 3.96, which rounded to the nearest whole number is 4 months. So the solution is 2 years and 4 months.

Example two requires us to convert 3.75 days into days and hours. To do this we multiply the 0.75 part of the decimal number by 24 hours to give 18. So the solution is 3 days and 18 hours.

Example three requires us to convert 7.33 hours into hours and minutes. To do this we multiply the 0.33 part of the decimal number by 60 minutes to give 19.8, which rounded to the nearest whole number is 20 minutes. So the solution is 7 hours and 20 minutes.

Example four requires us to convert 18.25 minutes into minutes and seconds. To do this we multiply the 0.25 part of the decimal number by 60 seconds to give 15. So the solution is 18 minutes and 15 seconds.

Activity 3: Practice Questions

Click on the Activity 3 link in the right-hand part of this screen.

Now have a go at converting decimal numbers to more practical forms on your own by working through some practice questions.
End of Topic

Congratulations, you have completed this topic.

You should now have a better understanding of Decimal Numbers.